Solar Energy 147 (2017) 300–313

Contents lists available at ScienceDirect

Solar Energy

journal homepage: www.elsevier.com/locate/solener

Islanded microgrid energy system parameter estimation using stochastic methods

Mitchell Lennard *, Abhijit Date, Xinghou Yu

RMIT University, Australia

ARTICLE INFO

Article history:
Received 22 July 2016
Received in revised form 5 December 2016
Accepted 20 December 2016

Keywords:
Microgrid energy systems
Optimisation
Stochastic programming

ABSTRACT

The Design Optimisation of small energy Microgrids involves establishing the equipment configuration (size and capacity of components) and the operational rules for ongoing in service use. Existing techniques support the analysis of individual days of incident solar energy. In this paper a technique that uses Stochastic programming is proposed. The new technique allows the probability of occurrence of a particular day of incident solar energy, and the probability occurrence of particular consecutive days of incident solar energy to be considered by the optimisation process. The technique outlined supports the pre processing of incident solar energy and system loads as discrete probability functions which subsequently allows simple linear programming techniques to be utilised. This results in the ability to create easy to modify and easily scalable design tools.

1. Introduction

Islanded Microgrids are a special form of Distributed Energy System. Here they refer to electrical power systems that are not connected to a larger electrical distribution network or grid. Distributed Energy (DE) Systems provide a technically and economic efficient architecture for the incorporation of renewable generation technologies into the electrical power supply system. DE systems are an effective early action greenhouse gas mitigation option for Australia when it is considered within a portfolio of other mitigation options and further Islanded operation of distribution networks is in principle highly effective in realising the full value from embedded generation (Lilley et al., 2009; He et al., 2008; Shenai and Shah, 2011).

This paper uses a highly simplified 'reference system' to explore issues surrounding the design optimisation of Islanded Microgrids. The refence system is shown as Fig. 1.

The optimisation of Microgrid energy systems involves both the analysis of the energy system configuration during the design phase and then the optimisation of system operation once it starts to produce energy.

For the reference system the key variable that impacts on the optimum design solution is the amount of incident solar energy on a given day. For an Islanded energy system comprised of a photovoltaic (PV) array, a diesel engine driven generator and a storage battery and for any given day of incident energy, the design optimisation involves increasing the size of the PV array and battery (increased capital cost) while reducing the size (capital cost) and running time (daily cost) of the generator. There will be an 'optimal' solution (size of PV array, battery, generator and generator running time) that produces the lowest Cost of Energy (COE). That optimal solution will only be valid for days with the same cumulative incident solar energy and electrical load requirement (where the load has partial weather dependency). On days with different incident energy characteristics the solution will not be optimal.

1.1. Existing approaches

An initial review of existing work addressing the modelling of energy systems, and energy system optimisation showed that the work could be separate categories:

Work focused on optimisation techniques where the energy systems are an example application. This category or work was found to be useful in exploring the relationship between objective function formulation and mathematical technique. This work was focussed on expanding mathematical techniques.

Work focussed on solving problems with the optimisation of energy systems. This body of work is focussed on investigating the detailed modelling of energy systems and tends to focus on defining the optimisation problem.

* Corresponding author.
E-mail address: Mitchell@Rickettspointpower.com (M. Lennard).

http://dx.doi.org/10.1016/j.solener.2016.12.041
0038-092X/

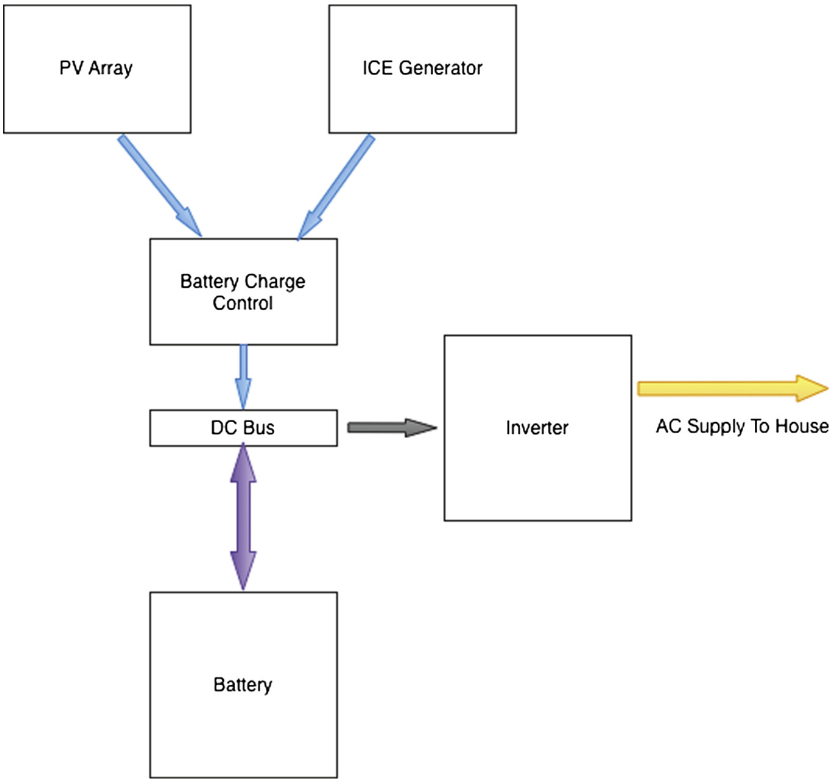


Fig. 1. Simple system being examined.

1.1.1. General optimisation with energy systems as an example

Assessment of energy system sizing is addressed a range of papers where the focus was found to be the exploration of mathematical techniques with the use of Microgrid optimisation as an example. There are a large number of these papers and in 2012 they where reviewed by Erdinc and Uzunoglu (2012). This review paper categorises work according to the mathematical approach adopted. Key mathematical categories looked at included Genetic Algorithms (GA), Particle Swarm optimisation (PSO), Simulated Annealing (SA), design space based approaches, simulation approaches, evolutionary algorithms (EA), stochastic/probabilistic approaches. It is noted that this review paper takes a very generic view of the nature of small energy systems and it is only when such papers are examined in detail is it possible to gain a view of the effectiveness of any particular approach.

Evolutionary strategies are found to suite energy system problems where the technology models are non linear or otherwise complex. Logenthiran et al. (2010) provides an example of this class of paper. This paper establishes a simple objective function of the form

$$MinC_T = \sum_{i=1}^{N} C_{DERi}N_{DERi} + OMC \text{ where}$$

C_T is the total cost of the distributed energy resources, C_{DERi} is the capital cost of DER_i and OMC is the ongoing operational and maintenance costs.

This simple objective function is subject to simple constraints of the form

$LPSP_{max}$ where

$LPSP$ is the probability of the loss of power supply.

Complexity is introduced by the system component modelling which introduces non-linear relationships to model wind plant and photovoltaic plant. This combination of simple objective functions and constraints together with complex plant models is a problem definition that suites the Evolutionary Strategy (ES) approach explored. The focus of the paper is on processing efficiency (and convergence) as much as it is on the Distributed Energy design issue. The ES approach explored does support the analysis of different system configurations (by amending the form and content of the ES 'chromosome'). Further because the Genetic Algorithm concept allows the systematic review of a large number of scenarios this paper can look at the performance of systems over an entire year (while processing on a day by day 24 h period).

This approach of using Genetic Algorithms is explored in depth by Bustos et al. (2012) in a paper which introduces two objective functions (Expected Energy not Supplied and Levelised Cost of Energy) together with a wide range of generating sources. This paper also introduces the concept of using Weibull distributions to estimate solar radiation and wind speed. This paper shows that scalability of using a Genetic Algorithm approach but is limited to analysis of systems over a 24 h period.

In (Kayal and Chandra, 2014) Kayal introduces probability estimation of wind and solar incident energy is combined with complex technology models and objective functions for the minimisation of annual average power loss and maximisation of power stability and network security indices. The combination of the probabilistic estimation of generator output with complex technical models, load requirements and system constraints results in a question that is best suited to some form of GA approach. In this example a form of Particle Swarm Optimisation (PSO) is adopted. This approach is suitable for the form of the problem developed, especially given the nature of the objective function, but it can be seen that it would not support a assessment of consecutive day performance, since the difference between consecutive days is hidden in the probability distribution created. As a consequence the answer produced is shown as an 'average' of system performance, in this case 4 day classes (Winter, Spring, Summer, Autumn).

Another class of papers looks at the concept of 'control strategies' as an approach to microgrid optimisation, Wang et al. (2012) provides an example of these approaches. In this paper a simulation style strategy, that systematically checks all possible combination of variables (for a given incident energy density, wind speed load requirement) and optimises Net Present Cost and renewable 'market penetration'. This approach is possible due to the use of a pre-determined 'control strategy'. In the context of these style of system this control strategy functions like a series of constraints. These control strategy constraints are calculated in an iterative process then become input conditions to the second stage of the GA analytical optimisation. The use of a pre-determined control strategy greatly reduces the number of possible system configuration options, making the GA analytical approach possible. The approach would support a form of sensitivity analysis where by it would be possible to change the control rules and examine the impact on the result. The approach is limited by the manner of its incorporation of incident radiated energy, wind speed and load all of which must be pre-declared for a given time increment.

Another class of papers examine the concepts of Robust Optimisation as applied to energy systems. As energy systems have inputs that are often uncertain (e.g. incident solar energy, wind speed, connection time of electric vehicles) they are suited to use as exemplars of Robust Optimisation techniques. In (Battistelli et al., 2012) Battistelli provides an example of this approach. This paper examines grid connected wind generators with battery storage provided by Electric Vehicles (EV). The approach adopted creates simple objective functions, with power balance and power flow constraints. These objective constraints are then amended by inclusion of terms representing 'auxiliary variables' and 'deviation' of certain parameters (in this case power transferred in and out of the Garage) The deviation parameters accommodate the uncertainty in the time EV's spend connected to the system and the auxiliary parameters allow related variations in generation and load parameters. Importantly this approach does not address the probability of a particular deviation occurring but rather establishes the most optimum solution assuming that the uncertainty variable can occur anywhere within the range set by the deviation constraint. As such this approach looks like a form a sensitivity analysis. The paper reaches conclusions about those parameters are sensitive to EV connection time and those that are not. In this way this approach does allow an insight into design decision making.

A further variation in technique that attempts to address the uncertainty inherent in incident solar energy is described by Cabral et al. (2010) which uses a Markov style analysis to estimate the incident solar energy at any time using some limited historical data. The Markov approach is used to estimate a daily 'clearness index' that is then used to estimate PV generation. A similar approach is used to estimate the state of the storage battery charge. These two inputs are then used analytically (operating rule approach) to optimise an objective function that addresses "Loss of Power Supply Probability" (LPSP). Importantly this work provides test evidence that allows a statistical review of the Markov estimation approach, demonstrating that it is useful in estimating and accommodating the uncertainty in incident solar energy.

The final paper reviewed in this group of work (Baziar and Kavousi-Fard, 2013) brings together a number of the concepts previously discussed into a Θ-PSO based optimisation that addresses total operating cost of a hybrid solar, wind, storage Micro Grid. The paper proposes a simple capital cost, running cost style objective function that could be simply expanded to accommodate a range of architectures. The constraints are related to both energy generation balance and a novel approach to battery charge rate limits. The method incorporates stochastic uncertainty using a double sided Point Estimate Method (PEM). The methodology solves the optimisation for the mean value of each random variable and then twice again for points above and below the random variable mean. The basic optimisation process used is Particle Swarm Optimisation (PSO). The paper proposes some sub process modifications to the PSO approach. The approach is limited to a 24 h period and assumes the random variables in question are Normally distributed.

The common aspect of all the work described in this section is that they primarily use the investigation of energy systems as a way to illustrate particular optimisation mathematical constructs. This does not mean that the insights provided are not useful, but what is illustrated is that most approaches use both objective function constructs and technology representations that are either simplified, or in some cases made more complex in order to suite the mathematical technique being investigated. Without exception all of the techniques investigated investigate a single 24 h period.

1.1.2. Energy system specific optimisation

A small number of examples of optimisation papers that are focussed on energy system optimisation as an outcome have been identified.

An early contribution is described by Dufo-Lopez and Bernal-Agustin (2008). This paper describes a method for Multi Objective Optimisation (MOO) of a PV-Wind-Diesel-hydrogen-battery system hybrid energy system. The optimisation addresses optimisation of total life cycle cost, un-met load and CO_2 emissions. This paper applies the know technique of Multi-Objective Evolutionary Algorithm (MOEA) modified by using the concept of a Strength Pareto Evolutionary Algorithm (SPEA). The approach adopted is to use a two linked evolutionary algorithms (EA). The first EA (the Main Algorithm) that optimises the configuration (size of each element) followed by a second algorithm that optimises 12 'control variables' (running times, state of charge, hydrogen tank discharge, etc.). The approach is computationally intensive but does allow the ability to incorporate complex constraints and system interactions (such as battery life vs. cycles). The approach supports whole year analysis but still uses limited weather and load variability. The main concern with this approach is the time taken to compute solutions that are still only a 'Pareto Frontier'. The advantages of this approach is that it does examine a full year of data and it does support the use of sensitivity analysis during design investigation.

Zhang et al. (2012) describes the Multi Objective Optimisation (MOO) analysis of a small Combined Heat and Power (CHP) micro-grid energy system. The analytical approach is developed to examine not only the trade-off between the cost of energy and the contribution of the systems operation to CO_2 emissions but also allows an examination of different generating (electricity and heat) technologies. A contribution of this paper is the use of 'nested' objective functions that allows existing Mixed Integer Linear Programming (MILP) tools (in this case the CPLEX solver supported by the GAMS tool-set). The approach adopted illustrates the concept of tailoring the structure of constraints, in this case generator ramp limits, energy demand constraints, CHP life constraints and thermal storage constraints to support the specific analysis questions (in this case what mix of generating technologies is optimal).

This paper introduces a second concept of incorporating total life cycle CO_2 emission, rather than just running emissions. This is achieved by amending the cost objective functions with fixed indices (GWP for CO_2 and AP for SO_2). These indices are developed suing a pre-existing Life Cycle Analysis (LCA) tool.

One major stream of work in the area is the use of complex simulation as the basis of the optimisation method. The two key contributions in this area are the Hybrid 2 Simulation model (Manwell et al., 2006) and the much used HOMER optimisation tool (Lambert et al., 2005). Hybrid 2 and Homer both use the concept of time step by step simulation of a defined energy system performance. Hybrid 2 is predominantly a simulation that could be used to support optimisation. HOMER uses a simpler form of the Hybrid 2 style simulation and then adds an optimisation process layer and sensitivity analysis layer which provided designers with data that can be used to identify optimal system configurations and operational rules. The HOMER and Hybrid 2 simulations have the following key characteristics:

- They use time series simulations that, for each time step, model the load requirement, the generation capacity, distribute energy to loads or storage using pre-defined rules sets (constraints) then produce metrics to analyse how effectively system aims have been addressed.
- They accommodate incident energy using a time step by time step estimate that could be historic data or an estimate based on historic data and probability distributions.
- They accommodate load requirements using a time step by time step estimate.
- They use the concept of the time step load requirement as the basis of all modelling. The rule sets and constraints are all developed to look at how that load requirement will be meet in that time period, whether or not energy needs to be 'imported' and what best to do with any energy that is generated (by renewable resources) but not required to meet that time step load.
- Each uses detailed 'models' of the behaviour and performance of system components. These models could be modified without impacting on the overall structure of the analysis tool....

The HOMER tool uses the time step simulation to then conduct an optimisation by investigating a range of configurations in predefined steps (e.g. increasing PV size in 5 kW steps). Configurations and then ranked according to pre-defined criteria (levelised cost of energy, fuel use, CO_2 emission). The sensitivity analysis is the same process of investigating the changes in yearly performance that result from changes in 'sensitivity parameters' rather than physical configuration. It would be possible to use the Hybrid 2 simulation as the basis of a similar optimisation approach as HOMER.

What both tools illustrate is that because they view optimisation as a design task (i.e. a task to establish system configuration and operation), rather than a mathematical rigorous, continuous optimisation, they can use a stepwise approximation/simulation methodology and achieve a suitable result. A designer could use the functionality in either of these to 'manually' amend design configurations to close in on an approximate optimal solution.

The DER-CAM tool-set (Marnay et al., 2001, 2003) represents a comprehensive optimisation tool that is not simulation based but uses an analytical mathematical approach. The DER-CAM tool set has evolved over time and includes two main versions: an Investment and Planning tool set and an Operations tool set. The present capability of DER-CAM is exemplified by papers such as Marnay et al. (2013, 2014). DER-CAM is primarily a tool to assess energy generation and energy balance in CHP systems with the aim of establishing the optimal configuration and operation in order to minimise energy costs and CO_2 emissions.

While DER-CAM mathematical constructs may become complex (Marnay et al., 2013, 2014) the basic approach initially adopted and subsequently maintained is to structure all objective functions as a simple linear relationship with the following form

$$minf = C^T * x = C_1 * x_1 + \cdots C_n * x_n$$

such that

$Ax < b$ and,

$L \leqslant x \leqslant U$ where
C is the cost coefficient vector
x is the decision variable vector
A is the constraint coefficient matrix b is the constraint coefficient vector
L is the decision variable lower bound and
U is the decision variable lower bound

By maintaining this structure and capturing the energy system characteristics and external variables as a series of constraints and the energy system configuration as decision variables DEC-CAM can use a Mixed Integer Linear Programming (MILP) solution. In the DER-CAM case this solution is mechanised using the CPLEX solver in the General Algebraic Modelling System (GAMS) tool. The basic DER-CAM structure limits the tool to examining energy flows in a discrete series of pre-defined time steps with the input energy and customer load defined for each of those time steps.

The potential for the basic DER-CAM structure to deal with uncertainty is explored in Marnay et al. (2014), which addresses uncertainty in Electric Vehicle (EV) connections and Siddiqui et al. (2004) which treats imported energy costs in a similar manner.

1.2. The opportunity to improve on existing approaches

The analytical tools examined above accommodate aspects of the optimisation question, either for initial configuration or identification of optimal operating rules.

For the reference energy system (Fig. 1) a key issue relates directly to the question of how weather variation over consecutive days impacts on the validity of the optimal solution. HOMER allows for a daily/monthly/yearly weather profile to be used in the simulation but for a given location this will always be an estimate. DERCAM and HOMER would both support a designer who wished to conduct a sensitivity analysis with weather as the sensitivity variable but none of the existing tools are structured directly around the concept of weather variability over consecutive days.

As noted earlier a design that is optimal for a given day of incident energy may not be optimal for days of less energy, especially for the case where there is a series of consecutive days of less incident energy. A design that is optimal for a given class or classes of days (which is how both DERCAM and HOMER are structured) day may need to be further amended to consider/accommodate the impact consecutive days of poor weather and low incident energy.

As outlined in the introduction for the simple system shown in Fig. 1 there will be an 'optimal' solution (size of PV array, battery, generator and generator running time) that produces the lowest Cost of Energy (COE). That optimal solution will only be valid for days with the same cumulative incident solar energy and electrical load requirement (where the load has partial weather dependency). On days with different incident energy characteristics the solution will not be optimal.

If the day ($n + 1$) has **less** incident energy than the analysed day (n) then there are two possible changes to the design solution. One is to add more (than is necessary for day n) PV and storage capacity (increase in capital cost) or add more generator running time on day $n + 1$ to compensate for the lower PV generation (running cost). This trade-off between capital cost and running cost represents a further optimisation question (beyond the day n optimisation) that is not addressed if the analysis is limited to 24 h. This problem is further exposed if day $n + 2$ again has less incident energy than day n. This issue is identified in Carpaneto et al. (2011).

In this paper a technique is proposed to address the multi-day question outlined using Stochastic Programming methods to explore optimal solutions, assuming a knowledge of the probability of occurrence of days of given incident energy.

2. Basic method

For the simple system (Fig. 1) the main variable of concern is the incident solar energy as this determines the amount of energy that can be generated. The size of PV arrays, electrical storage, is optimised, for a given load profile, based on the quantum of incident solar energy. The basic concept for the analysis proposed is to conduct a two stage optimisation. The first stage optimisation will be for the Modal day of incident solar energy for a given location over a 12 month period. Using the **Modal Day** as the starting point means that, by definition, the outcome of the first stage optimisation is valid for the greatest number of days in a given year. The second stage optimisation examines changes in the baseline solution based on the incident solar energy on non modal days, and the probability of those days occurring. This basic process flow is shown in Fig. 2.

A further consideration is which days should be examined. There are two basic situations:

a. The incident energy on day $(n + 1)$ is greater than the Modal Day (n) i.e. $Irad_{m(n+1)} > Irad_{m(n)}$. In this case the solar array will be larger than is necessary to provide the load and the battery will not be large enough to store all of the excess energy. This situation where excess energy is generated does not add to the overall levelised cost of energy (COE).
b. The incident energy on day $(n + 1)$ is less than the Modal Day (n) i.e. $Irad_{m(n+1)} < Irad_{m(n)}$. In this case the Modal day arrays size and storage will not be sufficiently large. The options available to address less than modal day incident energy are to either:
   - make up the shortfall in energy by importing non solar energy or
   - increase the size of the array and the storage (during the initial design and manufacturing phase) such that the energy shortfall on day $n + 1$ is captured and stored on day n.

Hence the new optimisation question, in words, is stated as follows:

> "*How to minimise the increase in capital cost (relative to the Modal day optimum configuration) versus the increased cost of imported energy for the days* $Irad_{m(n+1)} < Irad_{m(n)}$ *given that the a Modal day optimum exists and is known.*"

The technique, by looking at only less than modal days considers the question of the marginal cost of PV arrays vs generator running. If it is possible to reduce the PV array size and achieve a better annualized cost of energy this solution will be found by exploring only the less than modal days. If the optimisation that

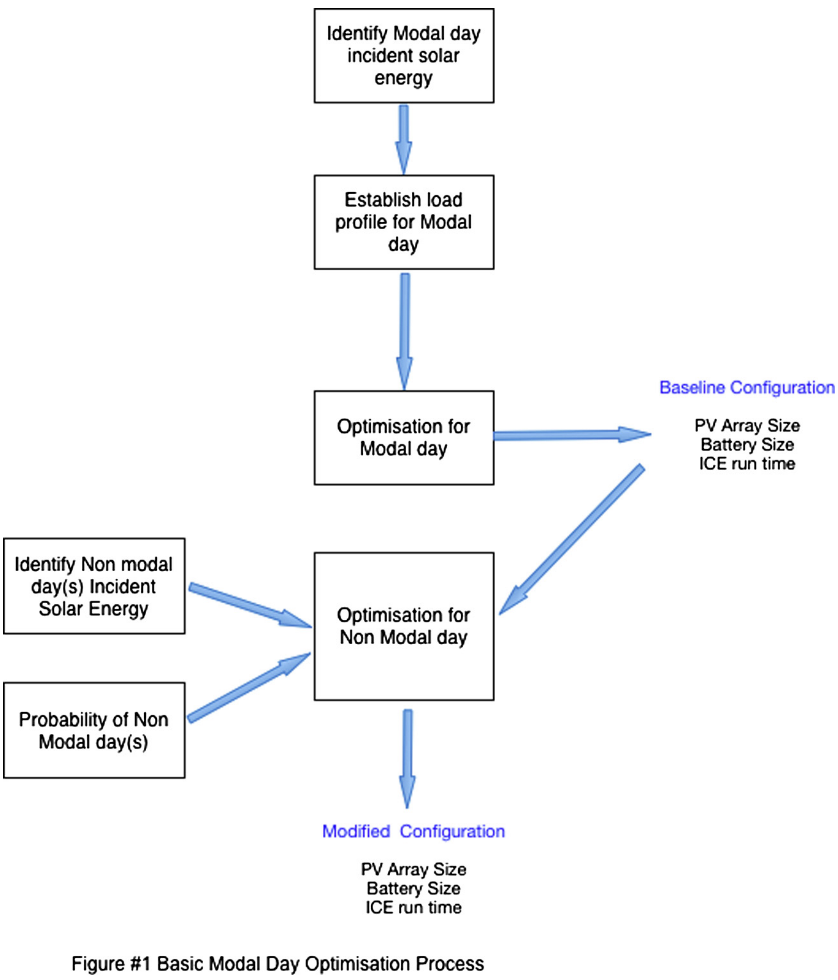


Fig. 2. Basic analysis process flow.

starts with the Modal day produces this result then it is possible to repeat the optimisation, starting with a greater than modal day. From an analysis perspective all that is required is that the analysis starts from some point, the Modal day is the most logical point but its not the necessary point. Choosing any incident energy day greater than modal then analysing the lesser incident energy days will produce the same result. This theory has been tested and found to be correct.

2.1. Incident energy as a discrete probability distribution

The basic concept is to develop a methodology that allows the impact of variations in consecutive days of incident Solar Energy and variations in load requirements on a day by day basis. In the method being used the scenarios are based upon the incident solar radiation. For this paper the analysis will use one year (2013) of recorded incident solar radiation data for Melbourne Airport, Melbourne Australia, as recoded by the Australian Bureau of Meteorology (http://www.bom.gov.au) (Australian Government Bureau of Meteorology, 2013).

While the incident solar energy is recorded as a linear record (at 0.1 kWh/m^2 resolution) it is possible to treat the incident energy as a series of discrete bands. As the energy systems are designed in discrete physical component sizes (I can have 10 PV panels or 12 panels or 13 panels, I have a 5 kWh or a 7.5 kWh battery) there will be a point where greater resolution of incident energy bands (smaller discrete bands of incident energy addressed in the analysis) will have no material impact on the design solution.

For this paper the width of the incident energy intervals is not crucial since the structure of the objective functions and constraint functions is independent of the total number of incident energy intervals. Consequently to support this preliminary exploration of the method the following simple discrete incident energy distribution will be used.

It is noted that in practice the 'width' of the incident energy bands addressed should be adjusted to match the sensitive of the design variables to the least design step.

A further consideration is the size of the sampled data used to create the Incident energy distribution. In this paper one year of daily incident energy data is processed. The risk with using one year of data is that the year chosen is atypical in some way. This issue can be addressed by creating the incident energy distribution using multiple years of data. For instance when 15 years of data (2000–2015) was processed for this same measuring site the modal incident energy day over 15 years was 1.84 kWh/m^2 and the distribution was different than that shown in Fig. 3. This ability to 'pre-process' weather data in this way is considered to be a key advantage of the methodology outlined.

In this example the single year of data is used, keeping in mind that this could just as easily been 15 years of data, the sample size has no impact on the basic form of the technique but will impact the 'accuracy' of the answer.

3. Stochastic programming with recourse as an analytical technique

The process described by Fig. 1 is able to be represented using the concepts of Stochastic Programming and general recourse optimisation. In (Higle, 2005) Higle describes the advantage of using Stochastic Programming with Recourse,rather than more simplistic approaches such as sensitivity analysis, for this style of problem. A small number of examples of the use of Stochastic Programming or the inclusion of uncertainty into energy Microgrid analysis have been identified. Handschin et al. (2006) and Siddiqui and Marnay (2008) show how to use stochastic methods to address questions regarding the design and operation of small energy systems. In (Chen et al., 2010) Fletena illustrates how stochastic programming methods can be applied to Hydro system water resource management. Stochastic programming methods are utilised by Chen et al. (2010) to provide an estimate of electrical energy power flows relative to wind variation a network with wind turbines. Cardoso adapts the DER-CAM tool using stochastic programming methods to model electrical vehicle storage in Cardoso et al. (2014) and generator reliability (availability) in Cardoso et al. (2013). There were no direct examples where multi-day optimisation of energy systems had included the ability to take account of incident solar energy uncertainty.

The general form of Stochastic Linear Program (SLP) is expressed as a two stage problem by Higle (2005) and King and Wallace (2010) using the following general form

$$Min C_1^T x_a + E_w Q(x_a, w)$$

Such that

$$Ax_a = b$$
$$x_a \geqslant 0$$

where

$$Q(x_a, w) = min d(w)^T y_i$$

such that

$$T(w)x_a + W(w)y_i = h(w)$$
$$y_i \geqslant 0$$

The first stage (processing step) aims to minimise the first stage known costs $C_1^T x_a$, plus the expected (recourse) costs $Q(x_a, w)$ over all possible scenarios, assuming that the first stage constraints $Ax_a = b$ are met.

The second stage (processing step) introduces a new set of variables, that can be used to minimise the cost for each second stage random scenario w. The constraint $T(w)x_a + W(w)y_i = h(w)$ links the first stage variables (chosen by the first linear program) and the amended equivalents of those variables impacted by the new second stage scenarios.

The two stage approach means that the first stage variables x_a are chosen independently of any future scenario(s) (*non anticipative property*) and each decision variable y_i depends upon the particular scenario that occurs. Hence the approach produces an optimum solution for the base scenario (x_a) and a series of supplemental solutions (y_i) one of which will be optimal given the occurrence of the random event w. In a 'system design' sense the first stage variables x_a can be viewed as initial design decisions and the second stage variables y_i are variations or deviations from the basic design that occur during operation as a result of the random events under consideration.

The consequence of this recourse model approach is not an absolute (single point) optimum solution but rather a series of solutions that are the 'least worse' that can accommodate the likely uncertainty.

As discussed by Wetts (1974) in order to process the two linear relationships it is possible, provided that w is a *discrete random variable*, to create a *Deterministic Equivalent* relationship (Wetts, 1974) which has the following form:

$$Min C_1^T x_a + \sum_{i=1}^{N} P_i d_i^T y_i$$

such that

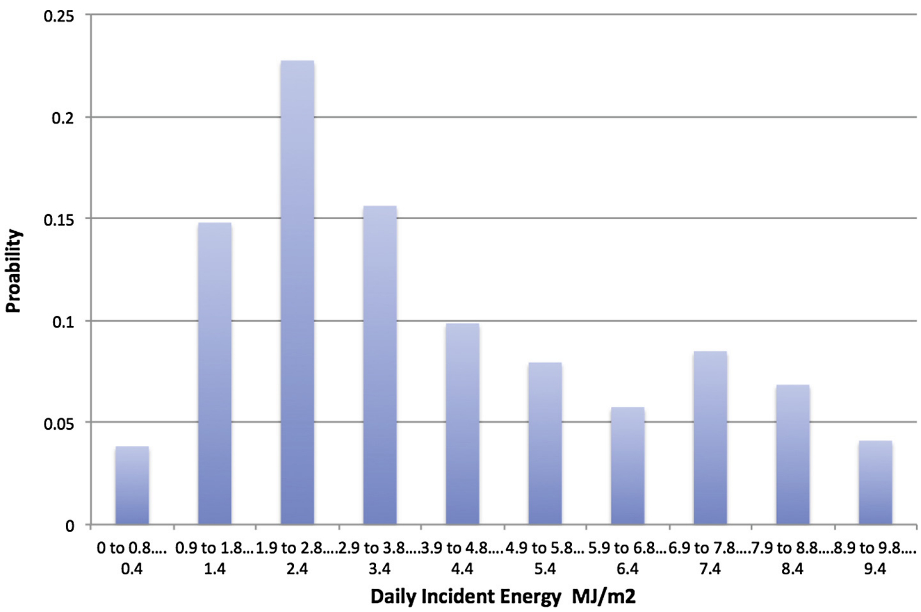


Fig. 3. Sample incident energy distribution - Melbourne Airport 2013 (Australian Government Bureau of Meteorology, 2013).

$$Ax_a = b$$
$$T_i x_a + W_i y_i = h_i \quad i = 1, \dots N$$
$$x_a \geqslant 0$$
$$y_i \geqslant 0 \quad i = 1, \dots N$$

where N is the number of scenarios and P_i is the probability of scenario i occurrence. Some observations regarding this deterministic form, relative to the energy system question being studied are:

- There is only one first stage x_a decision and there are i second stage y decisions.
- The first stage x_a decision cannot anticipate any one second stage scenario and must be feasible for each scenario, which means x_a is 'optimal' for all analysed scenarios.
- The T *and* W matrices are repeated for each scenario. So as the number of scenarios increase the size of the problem grows but not the structure of the linear program....

4. Analysis of a simple system

The application of the method described above is illustrated using the simple system shown in Fig. 1.

Note that while this example involves an islanded system this is not a necessary condition. Even in such a simple system the Internal Combustion Engine (ICE) could be replaced with a grid interconnection and the same optimisation questions regarding the incident radiated energy on days $n + i_{(i=1,2,3...)}$ would exist.

For the simple system the optimisation question is summarised by the following objective function:

$$minC_{elec} = C_{pv} + C_{ice} + C_{batt}$$

where

C_{elec} = the cost of electricity
C_{pv} = the cost of the PV generated electricity
C_{ice} = the cost of the generator generated electricity
C_{batt} = the cost of the battery storage

For this system the mathematical formulation is described as follows:

4.1. Stochastic equation forms - one day post modal

A common method for representing Stochastic Programming problems is to use scenario trees (Casey and Sen, 2005). If for the days $Irad_{m(n+1)} < Irad_{m(n)}$ we quantify the possible range of days of incident energy into discrete intervals $i = 1, 2 \dots n$ then the situation can be represented by the following scenario tree.

As the technique being explored is aimed at being a design tool it is necessary to declare those variables which represent 'initial design decisions' and those variables that depend on system running conditions. In this example the variables:

x_1 Size of PV array (m^2)
x_3 Battery Size (kWh)

are initial design decisions and the parameter

x_2 ICE Run-Time (hours per day)

is able to be varied at a later date once the probability and quantum of the reduced incident energy days is understood. In order to support the standard recourse problem form a new variable is declared.

y_{2i} ICE Run-Time (hours per day) for the post modal day

Transferring these defined variables back into the deterministic form shown as equation (He et al., 2008) gives:

$$minC_{elec} = ax_1 + bx_2 + cx_3 + \sum_i P_i by_{2i} + d \tag{3}$$

$\sum_i P_i by_{2i}$ is the arithmetic sum of each second stage scenario generator runtime as factored by the probability of that scenario, where $i = 1, 2, \dots n$ number of energy range days less than modal and
a, b and c are cost scaling factors for the PV capital cost, generator running cost and battery capital cost. While d is a factor covering all non variable fixed capital costs, which in this case includes the generator capital cost.

This can then be optimised using the following constraints:

4.2. Load constraints for the sample system

4.2.1. Modal day

The constraint is set such that on the Modal day the system is just able to supply all loads from the PV system, with no ICE run-time required. So:

$$P_{load(0)} = e_0 x_1 + f x_2 \quad \text{(a)}$$

where

e_0 is the incident energy (factored by PV array efficiency) for the modal day and,
f is the size of the system generator in kWh.

NOTE:

e_0 is a factor that includes a scalable performance metric for the PV array that would be determined by review of literature or by test.
f is chosen by the designer as a fixed element. In later more complex examples the reason to chose a fixed machine size (rather than make this a further variable) will be shown to be a 'constraint' associated with the ability to generate heat in a suitable time period.

A further constraint that aims to limit ICE use, and hence CO2 emissions (a form of Multi Objective Optimisation constraint) is to assert that all energy on the modal days should be derived from the PV system and hence state equation (a).

$$P_{load(0)} = e_0 x_1 \quad \text{(a)}$$

4.2.2. Non Modal day

On the Non Modal days the total load can be supplied by PV generation, ICE running or from extra energy stored on the Modal day. This leads to the following constraint:

$$P_{load(i)} = e_i x1 + f y_{2i} + energystoredintheModalday$$
$$P_{load(i)} = e_i x1 + f y_{2i} + (x_3 - g P_{load(0)})$$
$$P_{load(i)} + g P_{load(0)} = e_i x1 + f y_{2i} + x_3 \quad \text{(b)}$$

where

e_i is the incident energy (factored by PV array efficiency) for the non-modal day(s) and,
g is a factor that determines the percentage of the $P_{load(0)}$ that must be stored. This is a simple ratio style factor that accounts for the notion that the total incident energy for a day is not evenly distributed across 24 h but is available for direct load support fro only a few hours of any given day.

4.3. Battery constraints for sample system

From the Modal day, as an absolute minimum the battery must store at least a specific fraction g of the total daily load:

$$x_3 \geqslant P_{Load(0)} * g$$

Substituting (a)

$$0 \geqslant e_0 g x_1 + f g x_2 - x_3 \quad \text{(c)}$$

4.4. Stochastic equation forms - two days post modal

The above discussion provides a method to accommodate the idea that the incident energy on day $(n + 1)$ is less than the modal day (n) $Irad_{m(n+1)} < Irad_{m(n)}$. The next design driver is the concept that $Irad_{m(n+j)} < Irad_{m(n)},\ j = ,2,3\ldots k$. Note that this is the critical design case since its the total run of days less than modal that is important to the design approach, once any consecutive day has incident energy equal to or greater than the modal day $Irad_{m(n+j)} > Irad_{m(n)}$ then the capability of the system will reset and the optimisation that was valid for the modal day is valid again. There are two ways to deal with this case.

4.4.1. Option 1 multi-stage recourse

The scenario tree for this approach is illustrated below.

This scenario tree can be processed using a range of approaches (Higle, 2005). Mathematically robust approaches are suggested in Casey and Sen (2005) and Higle and Sen (1991). Another technique used is to work out the optimum solution and cost for each individual branch and then statistically combine the results then use statistical measure (such as the variance) to establish the most likely solution (Pishvaee et al., 2011). These approaches are mathematical viable but complex and result in very large processing requirements as the number of Non Modal incident energy bands increases.

4.4.2. Option 2 use of convolution

In the approach being proposed, while all days being analysed have less incident energy than the Modal day, the incident radiated energy on any day can be viewed as statistically independent from the preceding or following day. This suggests that simple convolution can be used to combine the three level scenario tree shown in Fig. 5 back to into a two level scenario structure as shown in Fig. 4. While the 'derived' scenario structure will have more nodes than the original $Irad_{m(n+1)} < Irad_{m(n)}$ scenario it will now be a single level structure able to be solved using the sample simple mathematical approach. This use of convolution has previously been explored in the analysis of hybrid wind/solar PV energy systems, where wind energy and solar radiance where assumed to be independent (Tina and Gagliano, 2011). The following scenario tree

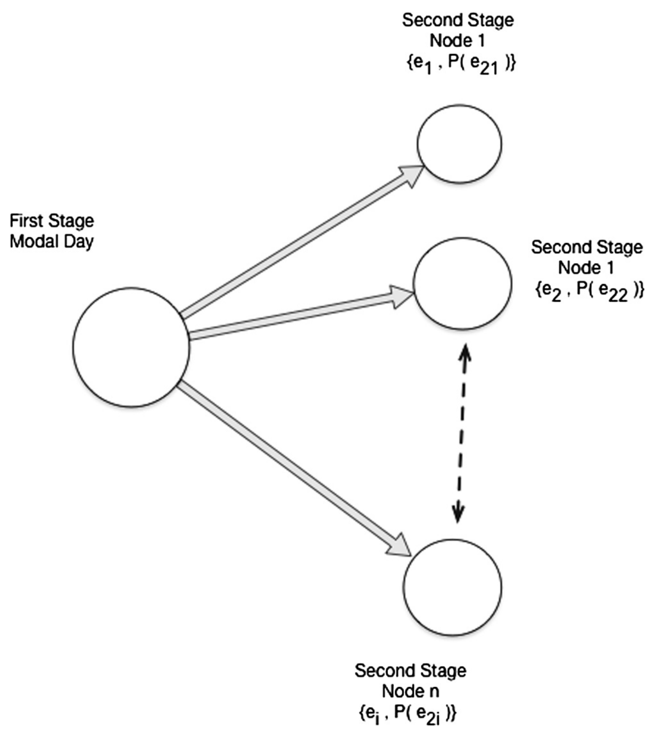


Fig. 4. One day post modal scenario tree.

The scenario tree for this approach is illustrated below

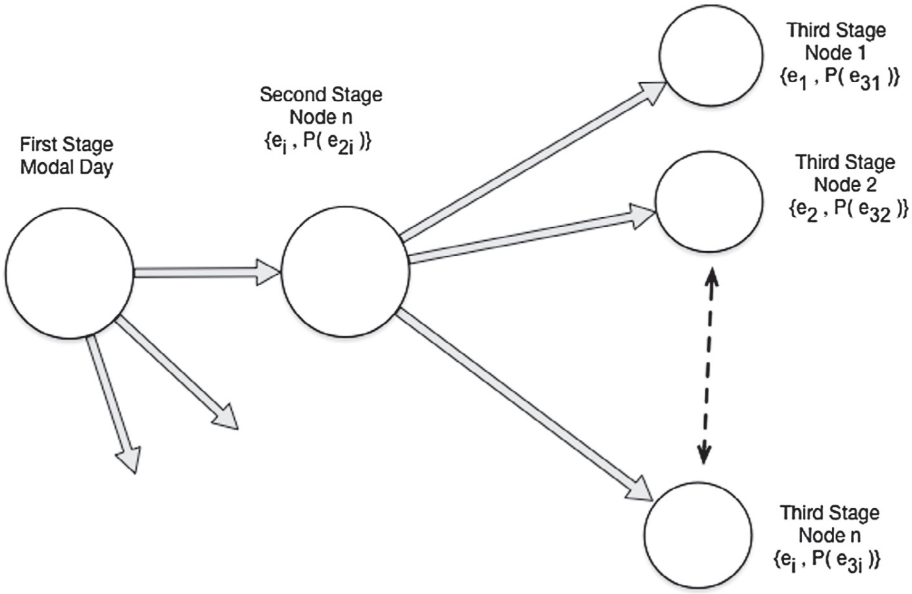


Fig. 5. Scenario tree for days $Irad_{m(n+j)} < Irad_{m(n)},\ j =, 2, 3 \ldots k$.

represents the situation $Irad_{m(n+j)} < Irad_{m(n)},\ j =, 2, 3 \ldots k$ (see Fig. 6).

where

$e_z = e_i + e_j$
$P_{e2z} = P_{e2i} x P_{e3j}$
$P_{Load(z)} = P_{load(i)} + P_{Load(j)}$

and the resultant objective function becomes the same form as the $Irad_m(n + 1)$ case

$$minC_{elec} = ax_1 + bx_2 + cx_3 + \sum_z P_z by_{2z}$$

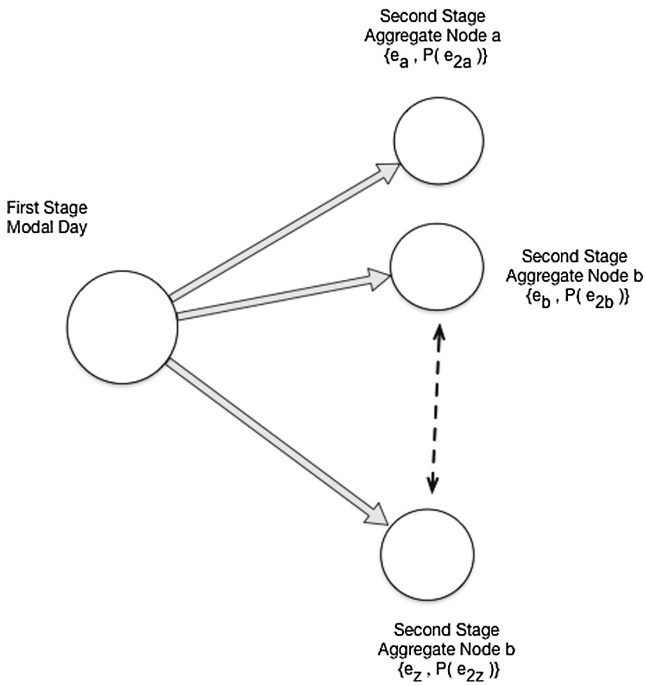


Fig. 6. Multi-day convoluted scenario tree.

with the resultant constraints being of the same form as previously discussed.

This simple convolution approach is possible because of the nature of the formulation of the original stochastic objective function and the underlying assumptions that the energy load in the less than incident energy days can only be meet by additional energy stored on the modal day, by sub modal day generator running time or a combination of both. Once this assumption is made then it is possible to combine strings of consecutive less than Modal days together provided that the total incident energy and total load for those days is aggregated.

5. Specific example and technical models

Earlier Eq. (3) provided the basic form of the objective function:

$$minC_{elec} = ax_1 + bx_2 + cx_3 + \sum_i P_i by_{2i} + d \qquad (3)$$

In optimisation approaches in general and specifically when using stochastic programming with recourse parameters are described as follows:

5.1. Decision parameters

In this example the decision parameters are

x_1 Size of PV array (m^2)
x_2 ICE Run-Time (hours per day)
x_3 Battery Size (kWh)

Note: this is true if the generator size is not a variable, which is the case in the process being developed.

5.2. Input parameters

The “Input Parameters” are used to establish the cost factors (a, b, c and d) in the objective function. These are in this example defined as follows:

$C_{batt/n}$ = Battery cost per kWh
$C_{pv/n}$ = PV cost per m^2
$ICE_{run/n}$ = Cost per hour (i.e. fuel cost)
ICE_{cap} = Capital cost (engine plus generator)
ICE_{gen} = ICE Generator Size (kW)
SL = System Life (years)

5.3. Input variables

The "input variables" are declared by the designer and are factors that are variable from a design perspective but are fixed in advance of the analysis.

P_{load} = Electrical load (kWh/day)
$Irad_m$ = Total Incident Radiation for Modal day (kWh/m^2)
μ_{pv} = PV Panel efficiency
t_{pvhrs} = assumed hours of sunshine where array produces hourly load
DoD_{spec} = DoD required to meet life estimate (manufactures estimate)
B_L = Battery Life
r = assumed interest rate

Note that the analysis approach assumes that the designer can vary the input parameters and input variables as a way to conduct a sensitivity style analysis. It would be possible to incorporate some of these variables into the optimisation but other important capabilities of the optimisation approach may be lost. Examples of how these variables can be captured in the optimisation, in the form of constraints, is explored in later chapters.

For this first example the following parameters are used:

5.4. System relationships

The factors a, b, c are used to translate the equipment characteristics shown in Fig. 3 into the daily equivalent costs that match the form of the objective function Eq. (3). Daily cost are used as the loads, and incident energy and probability of incident energy are all analysed on a per day basis. Costs for long-life capital purchases are calculated using "Equivalent Annual Cost" which is then reduced to a daily cost as follows:

$$EAC = InitialCost/A_{SL,r}$$

where $A_{SL,r}$ is the annuity rate

$$A_{SL,r} = [1 - 1/(1+r)^{SL}]/r \tag{i}$$

where SL is the system life in years and r is the assumed long term interest rate.

It was decided the cost comparison would be done on an COE per annum on a present year baseline. This is a highly conservative approach as it allows the COE for the CHP system to be compared with the COE from the competing grid energy, in baseline dollars. Across the course of the project the COE for the CHP system stays at the baseline dollar amount, so effectively drops each year, whereas the cost of grid energy notionally rises with inflation.

The Annuity Rate is a way to establish the total cost of the capital investment (today dollar capital cost plus interest charges assuming a payback equal to the system life) the distribute this equally across the system life years. (keeping in mind this is all in baseline dollars). This approach is used in both the Hybrid 2 and DERCAM model as way to distribute capital costs.

The second construct is where there is a component life that is less than the system life. In this case the baseline cost is factored up by the ratio. Again this is conservative since the initial interest charge will be higher (as its costed over a longer period) and then the mid life capital purchase will be paid in later year dollars but accounted for in today dollars.

$$a = C_{pv/n}/(A_{SL,r}) \times (SL/PL)] \quad \text{(This is an annualised cost per m}^2\text{)} \tag{ii}$$

$$b = 365xICE_{run/n} \tag{iii}$$

is the yearly fuel cost which assumes a defined generator/ICE combination

$$c = C_{batt/n}/(A_{SL,r}) \times (SL/BL) \quad \text{(This is an annualised cost per kWh)} \tag{iv}$$

Note: The battery life concept is simplified because the analysis assumes the battery size is made ideal for the MODAL day (see constraint equations) Moving away from the MODAL day changes (for some forms of battery chemistry) the battery life, and hence the total battery cost which is ignored at this point in the development of the basic relationships.

the factors e, f and g are ...

$$e_0 \tag{v}$$

is the incident energy (factored by PV array efficiency) for the modal day,

$$e_0 = Irad_m x \mu_{pv} \tag{vi}$$

Note μ_{pv} is a scaling factor that combines the specified efficiency of a given PV array together with a second factor that account for the variation in performance of the panel across the course of the day. This will be left as a simple scaling factor in this example and explored in a more representative fully detailed example in later chapters.

f is the size of the system generator in kWh (vii)

g is a factor that determines the percentage of the $P_{load(0)}$ that must be stored, that is the proportion of the load that occurs on the Modal day when the PV cannot generate.

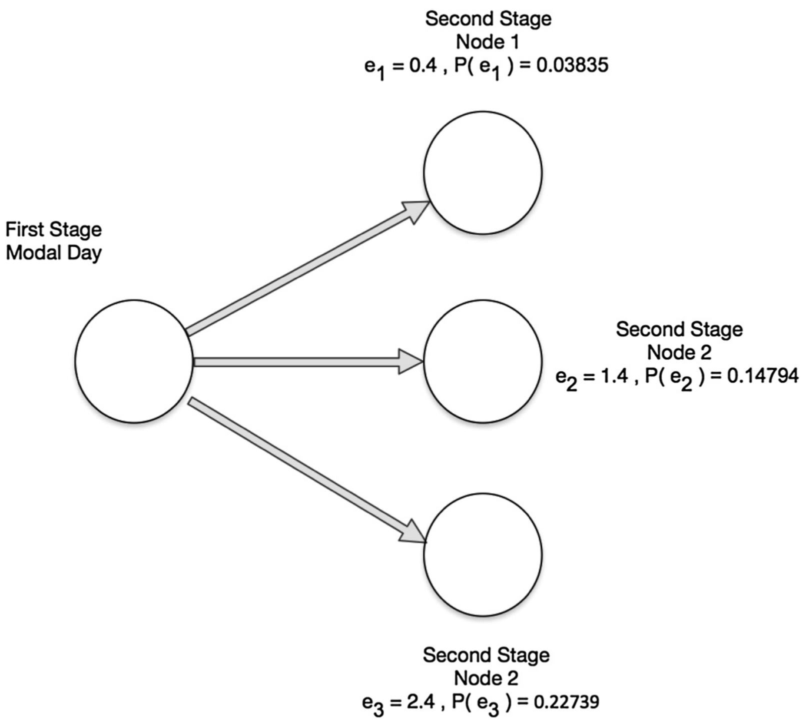
$$g = [(24 - t_{pvhrs})/24]/(DoD_{spec}) \tag{viii}$$

$$d = ICE_{cap}/(A_{SL,r}) \times (SL/MachineLife)$$

annualised capital cost of the ICE.

5.5. Specific weather scenarios

Fig. 3 shows the Incident Solar Energy daily probability distribution for Melbourne Airport in 2013. The Modal day incident radiant energy for this example was 2.8 MJ/M^2. This means that there are three classes of days with less incident energy than the modal day. This is represented by the following scenario tree:



6. Worked example

6.1. Cost function and the f matrix

The data from Table 2.2 is formatted into the defined equations as follows:

assuming an assumed interest rate of 6% and a design life of 20 years

$$A_{SL,r} = [1 - 1/(1+r)^{SL}]/r \quad \text{(i)}$$

$$A_{SL,r} = [1 - 1/(1+0.6)^{20}]/0.6$$

$$A_{SL,r} = 11.5 \quad \text{(ii)}$$

assuming $C_{pv/n} = \$250\ \text{m}^2$ then

Table 2.2
Input parameters.

Parameter	Description	Value	Comment
$C_{batt/n}$	Battery Cost per kWh	$1000	Based on LG batteries configured for small scale systems
$C_{pv/n}$	PV cost per m^2	$250	Based on Winacco WST260
ICE_{hr}	ICE Cost per hour (i.e. fuel cost)	$2.37	Estimated for Paguro machine @ $1.20 per L fuel cost
ICE	ICE Generator Size (kW)	5	Paguro P6000 6.5 kW machine
n_{pv}	PV panel efficiency	0.14	Based on Winacco WST260
SL	System Life (years)	20	Analysis variable
t_{pvhrs}	Assumed hours of sunshine where array produces hourly load	6	Analysis variable
DoD	DoD required to meet life (manufactures estimate)	80%	Manufactures spec
BL	Battery Life	10	Estimate based on one cycle per day to 80% DoD

$$a = C_{pv/n}/(A_{SL,r}) \times (SL/PL)]$$

$$a = 250/11.5 * (20/20)$$

$$a = 21.74$$

for the Paguro machine there is 0.35 l/kWh quoted fuel burn. It is 6.5 kW machine therefore one hour running = 2.275 l @ $1.20 per l = $2.73/h

$$b = 365 \times ICE_{run/n}$$

$$b = 365 \times 2.73$$

$$b = 997$$

assuming a $1000 per kWh battery and a 10 year battery life

$$c = C_{batt/n}/(A_{SL,r}) \times (SL/BL)]$$

$$c = 1000/11.5 \times (20/10)$$

$$c = 174$$

Based on the $11,000 cost of the P6000 machine

$$d = ICE_{cap}/(A_{SL,r}) \times (SL/MachineLife)$$

$$d = (11000/11.5) \times (20/20)\ d = 956$$

Based on the detail in the scenario tree

$$P_{e1} = 0.03385$$

$$P_{e1} = 0.17494$$

$$P_{e1} = 0.22739$$

Substituting back into the cost function, Eq. (3)results in the following:

$$minC_{elec} = ax_1 + bx_2 + cx_3 + \sum_i P_i b y_{2i} + d$$

$$minC_{elec} = 21.74x_1 + 997x_2 + 174x_3 + 0.03385 * 997y_{21} + 0.0.17494 * 997y_{22} + 0.22739 * 997y_{23} + d$$

$$minC_{elec} = 21.74x_1 + 997x_2 + 174x_3 + 33.74845y_{21} + 174.415y_{22} + 226.797y_{23} + d$$

This results in an f matrix as follows:

$$f_x = \begin{bmatrix} 21.74 \\ 997 \\ 174 \\ 33.748 \\ 174.415 \\ 226.797 \end{bmatrix}$$

6.2. Load constraints and the A_{eq} matrix

To compete the load constraints;

$f = ICE_{gen}$ which in this case is equal to 5 kW

assuming t_{pvhrs}, which is the total hours that the PV can supply load directly without storage, is 6 h, and assuming a Battery Depth of Discharge (DOD) limit of 80% then:

$$g = [(24 - t_{pvhrs})/24]/(DoD_{spec})$$
$$g = [(24 - 6)/24]/(0.8)$$
$$g = 0.9375$$

The PV conversion factors, see Eq. (iv), are defined as assuming a total conversion efficiency (μ_{pv}) of 0.14:

$$e_0 = Irad_m \times \mu_{pv}$$
$$e_0 = 2.8 \times 0.14 = 0.392$$
$$e_1 = 0.4 \times 0.14 = 0.056$$
$$e_0 = 1.4 \times 0.14 = 0.196$$
$$e_0 = 2.4 \times 0.14 = 0.336$$

Substituting back into Eqs. (a) and (b) results in the following equality constraints:

$$P_{load(0)} = e_0 x_1 + f x_2$$
$$P_{load(0)} = 0.392 x_1 + 5 x_2$$

and

$$P_{load(1)} + g P_{load(0)} = 0.056x1 + 5y_{21} + x_3$$
$$P_{load(2)} + g P_{load(0)} = 0.196x1 + 5y_{22} + x_3$$
$$P_{load(3)} + g P_{load(0)} = 0.366x1 + 5y_{23} + x_3$$

This results in A_{eq} being 4×6 matrix:

$$A_{eq} = \begin{pmatrix} 0.392 & 5 & 0 & 0 & 0 & 0 \\ 0.056 & 0 & 1 & 5 & 0 & 0 \\ 0.196 & 0 & 1 & 0 & 5 & 0 \\ 0.336 & 0 & 1 & 0 & 0 & 5 \end{pmatrix}$$

and the b_{eq} matrix is defined as:

$$b_{eq} = \begin{pmatrix} P_{load(0)} \\ P_{load(1)} + g P_{load(0)} \\ P_{load(2)} + g P_{load(0)} \\ P_{load(3)} + g P_{load(0)} \end{pmatrix}$$

Table 2.3
Simple system results.

Design parameter	Load = 20 kWh/day	Load = 40 kWh/day
x_1 = PV in m^2	51.02	102.4
x_2 = Gen run time	0	0
x_3 = Batt Size (kWh)	20.3	40.56
y_{21} Prob 0.034	3.37	6.74
y_{22} Prob 0.175	1.94	3.88
y_{23} = Prob 0.228	0.21	0.42

6.3. Storage constraints and the A matrix

As this is a simple system there is a single storage constraint as shown in Eq. (c)

$$0 \geqslant e_0 g x_1 + f g x_2 - x_3$$
$$0 \geqslant 0.392 * 0.9375 x_1 + 5 * 0.9375 x_2 - x_3$$
$$0 \geqslant 0.3675 x_1 + 4.6875 x_2 - x_3$$

This results in A being a 6×1 vector

$$A = (0.3975 \quad 4.6875 \quad -1 \quad 0 \quad 0 \quad 0)$$

and $b = 0$.

6.4. Simulation results

The one day post modal solution defined above was processed using the LINPROG function in Matlab and the load was set as equivalent on the Modal and Non Modal days. The following results where obtained.

The results in Table 2.3 show that the 'optimal' initial design decision regarding the size of the PV array varies when Non Modal days are factored into the solution using the Stochastic Programming methodology.

The results are interpreted as follows:

- The values for x_1 and x_3 are slightly greater than the minimum required to meet the model day constraints. This illustrates that these values have been amended by consideration of the non modal days.
- The values for y_{21}, y_{22}, and y_{23} represent the generator running time that will result if those less than modal days occur. There is a probability of 0.034 that on any given day a system with a 51.02 m^2 PV array and a 20.3 kWh battery will need to run the generator for 3.37 h.

In the literature discussion it was noted that "*The consequence of this recourse model approach is not an absolute optimum solution but rather a series of solutions that are the 'least worse' that can accommodate the likely uncertainty.*"

This example illustrates this concept. The design solution chosen for the PV size and battery analysis may not be the minimum possible (hence cheapest) solution for most common (modal day) but it does represent the least worse solution when the probability of non-modal days is considered.

7. Observations from the simple system stochastic solution

A number of capabilities not present in existing modelling approaches have been accommodated during the development of the simple stochastic model. It is not that this technique necessarily produces a better final answer than existing tools. Rather this technique allows certain aspects to be explored more easily than existing tools. Further the technique provides an estimate of the probability that particular running costs will be encountered across the life of the system. This estimate of running costs with an associated probability of occurrence is not supported by existing tools. The features that are incorporated into the technique are summarised as follows:

7.1. Incorporating weather variability

The form of the stochastic equations adopted allow the 'preprocessing' of weather data since the probable variation in incident solar energy (which is the key environmental variable renewable microgrids) is incorporated as the combination of a simple discrete input energy and a probability of occurrence of that energy. In this example the weather data is taken from a single year and has been divided into 10 discrete levels. This is an arbitrary breakdown. It would be possible to break down the data into smaller and smaller intervals which would result in a 'closer' to optimal solution. Using smaller energy intervals in this way does not impact on the simple form of the solution. Halving the energy intervals would result in the example case producing a 6 node 'less than modal' scenario tree which would be solved in the same way, with no change to either the form of the objective function or constraints.

Similarly it would be possible to use multi-years of historical data to produce a more robust prediction for future years. The method used to develop the incident radiation probability distribution is independent of the form of the stochastic equations. This is important since the quality of data available for different geographic locations varies greatly. Hence being able to separate weather data processing from the optimisation technique and to be able to pre-process weather using statistical techniques independent of the optimisation technique is a useful characteristic of this method.

Of particular interest is the transparency the technique provides of the impact of multiple consecutive Less than Modal days on the optimal solution. In the data set used for this example there is a cluster of strongly Less than Modal days for the months June, July and August. The results reported included these days into the overall yearly probability distribution. Having the ability to identify consecutive day incident energy patterns, and review the impact on the probabilities that flow to the stochastic equations allows decisions to be taken about what weather patterns to use in the design optimisation.

The ability to post process weather data and examine the impacts of consecutive day weather variation is considered a key advantage of the technique described.

7.2. Incorporating load data

Load data is incorporated as discrete daily totals in the constraint equations. As with the weather data this allows 'pre-processing' of the load requirements which in turn keeps the form of the optimisation solution simple. In this example the concept of a daily load profile (load v.s. incident energy v.s. time) is greatly simplified. It is possible using the form of the equations presented to incorporate the variation in load profiles by adjustment of the pre-calculated scaling factors in the constraint relationships. The form of the constraints also allows variation of load profile in each Non Modal day to be incorporated while leaving the core processing technique unchanged.

7.3. Incorporating system complexity

The form of the solution presented is able to scale up to incorporate increased system complexity. Adding system components (e.g. Hot water generation and hot water storage) will increase the terms in the objective function without increasing the complexity of the mathematical technique required to find a solution. The relationships between hot water and electrical storage, and hot water and electrical load will be expressed as an increased in the range of constraint relationships. While this will lead to larger solution matrices the core technique, and the ability to use simple solvers will not be made more complex. Likewise the ability to accommodate weather variation using the stochastic method is not impacted by the increase in system complexity.

7.4. Basis of a transparent design tool

A further advantage of the technique outlined is that the structure of the objective function and constraint equation provides the basis of an easy to use design tool where the impact of assumptions on the final result is clear to the user. The structure of the solution would allow designers to conduct transparent sensitivity analysis using simple equation solvers.

7.5. Transparent commercial risk

The method proposed provides an opportunity to not only assess the variable costs (ICE run time or imported energy) associated with a given design solution but the probability that those costs will be incurred. This is a measure of potential commercial risk and is a unique capability of the approach being adopted.

8. Conclusion

There is an advantage to be able to accommodate the probable consecutive day weather variations when establishing optimal design configurations for Microgrid energy systems. Considering consecutive days allows the cost of energy gathered and stored from renewable sources on day n to be traded off against importing energy or using local ICE generation to cover renewable generation deficiencies on day $n+i,\ i=1,2,3,\ldots$ The approach examined uses the concept the 'Modal' incident energy day as a starting point to apply Stochastic Programming methods which allows a probabilistic consideration of days with incident energy less than the Modal starting point. The method examined results in a form of Stochastic equations that allow the use of simple linear solvers while at the same time accommodating variations in weather, load and system complexity.

References

Australian Government Bureau of Meteorology – Climate Data Online – Update 13 December 2013.

Battistelli, C., Baringo, L., Conejo, A., 2012. Optimal energy management of small electric energy systems including V2G facilities and renewable energy sources. Electr. Power Syst. Res., 50–59

Baziar, A., Kavousi-Fard, A., 2013. Considering uncertainty in the optimal energy management of renewable micro-grids including storage devices. Renew. Energy, 158–166.

Bustos, C., Watts, D., Ren, H., 2012. MicroGrid operation and design optimization with synthetic winds and solar resources. Latin Am. Trans., IEEE (Revista IEEE America Latina), 1550–1562.

Cabral, C., Filho, D., Diniz, A., Martins, J., Toledo, O., Neto, L., 2010. A stochastic method for stand-alone photovoltaic system sizing. Sol. Energy, 1628–1636.

Cardoso, G., Stadler, M., Siddiqui, A., Marnay DeForest, C., Barbosa-Povoa, A., Ferrao, P., 2013. Microgrid reliability modeling and battery scheduling using stochastic linear programming. J. Electr. Power Syst. Res.

Cardoso, G., Stadler, M., Bozchalui, M.C., Sharma, R., Marnay, C., Barbosa-Povoa1, A., Ferrao, P., 2014. Optimal investment and scheduling of distributed energy resources with uncertainty in electric vehicles driving schedules. Energy 64, 17–30.

Carpaneto, E., Chicco, G., Mancarella, P., Russo, A., 2011. Cogneration planning under uncertainty Part 1: multiple time frame approach. Appl. Energy 88, 1059–1067.

Casey, M.S., Sen, S., 2005. The scenario generation algorithm for multistage stochastic linear programming. Math. Oper. Res. 30, 615–631.

Chen, P., Siano, P., Bak-Jensen, B., Chen, Z., 2010. Stochastic optimisation of wind turbine power factor using a stochastic model of wind power. IEEE Trans. Sustain. Energy 1.

Dufo-Lopez, R., Bernal-Agustin, A., 2008. Considering uncertainty in the optimal energy management of renewable micro-grids including storage devices. Renew. Energy, 2559–2572.

Erdinc, O., Uzunoglu, M., 2012. Optimum design of hybrid renewable energy systems: overview of different approaches. Renew. Sustain. Energy Rev., 1412–1425

Handschin, E., Neise, F., Neumann, H., Schultz, R., 2006. Optimal operation of dispersed generation under uncertainty using mathematical programming. Int. J. Electr. Power Energy Syst. 28, 618–626.

He, M.M., Reutzel, E.M., Jiang, X., Katz, R.H., Sanders, S.R., Culler, D.E., Lutz, K., 2008. An architecture for local energy generation, distribution, and sharing. In: Energy 2030 Conference, 2008. ENERGY 2008. IEEE, pp. 1–6, 17–18.

Higle', J.L., 2005. Stochastic programming: optimisation when uncertainty matters. In: Informs. ISBN: 1-877640-21-2.
Higle, J., Sen, S., 1991. Stochastic decomposition: an algorithm for tow-stage linear programs with recourse. Math. Oper. Res. 16, 650–669.
Kayal, P., Chandra, K., 2014. Optimal mix of solar and wind distributed generations considering performance improvement of electrical distribution network. Renew. Energy, 173–186.
King, A.J., Wallace, S.W., 2010. Modeling with Stochastic Programming Springer series in Operations Research and Financial Engineering. ISBN 978-0-387-87817-1.
Lambert, T., Gilman, P., Lilienthal, P., 2005. Micropower system modeling with HOMER. In: Farret, F.A., Simões, M.G. (Eds.), Integration of Alternative Sources of Energy. John Wiley & Sons. ISBN 0471712329.
Lilley, B., Satzow, A., Jones, T., 2009. A Value Proposition for Distributed Energy In Australia. CSIRO Report.
Logenthiran, T., Srinivasan, D., Khambadkone, A.M., Raj, T.S., 2010. Optimal sizing of an islanded microgrid using Evolutionary Strategy, Probabilistic Methods Applied to Power Systems (PMAPS). In: 2010 IEEE 11th International Conference, June 2010, pp. 12–17.
Manwell, J., Rogers, A., Hayman, G., Avelar, C., McGowan, J., Abdulwahid, U., Wu, K., 2006. Hybrid 2 A Hybrid Systems Simulation Model Theory Manual. Renewable Energy Research Laboratory, Department of Mechanical Engineering University of Massachusetts, National Renewable Energy Laboratory.
Marnay, C., Chard, J., Hamachi, K., Lipman, T., Moezzi, M., Ouaglai, B., Siddiqui, A., 2001. Modelling of Customer Adoption of Distributed Energy Resources. Lawrence Berkely National Laboratory. LBNL-49582.
Marnay, C., Siddiqui, A., Firestone, R., Ghosh, S., Edwards, J., Stadler, M., 2003. Distributed Energy Resources with Combined Heat and Power Applications. Lawrence Berkely National Laboratory. LBNL-52718.
Marnay, C., Siddiqui, A., Stadler, M., DeForest, N., Donadee, J., Bhattacharya, P., Lai, J., 2013. Applications of Optimal Building Energy Systems Selection and Operation. Lawrence Berkely National Laboratory. LBNL-6302E.
Marnay, C., Stadler, M., Cardoso, G., Bozchalui, Sharma, R., Barbosa-Povoa, A., Ferrao, P., 2014. Optimal Investment Scheduling of Distributed Energy Resources with Uncertainty in Electric Vehicles Driving Schedules. Lawrence Berkely National Laboratory. LBNL-6471E.
Pishvaee, M., Rabbani, M., Ali Torabi, S., 2011. A robust optimisation approach to closed loop supply chain network design under uncertainty. Appl. Math. Modell. 35, 637–649.
Shenai, K., Shah, K., 2011. Smart DC micro-grid for efficient utilization of distributed renewable energy. In: 2011 IEEE Energytech, pp. 1–6, 25–26.
Siddiqui, A.S., Marnay, C., 2008. Distributed generation investment by a microgrid under uncertainty. Energy 33, 1729–1737.
Siddiqui, A.S., Marnay, C., Bailey, O., LaCommare, K.H., 2004. Optimal selection of on-site generation with combined heat and power applications. Int. J. Distrib. Energy Resour. 1 (1), 33–62.
Tina, G.M., Gagliano, S., 2011. Probabilistic modelling of hybrid solar/wind power system with solar tracking system. Renew. Energy 36, 1719–1727.
Wang, Chengshan, Liu, Mengxuan, Guo, Li, 2012. Cooperative operation and optimal design for islanded microgrid. In: Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES, Jan 2012 pp. 1–8.
Wetts, R., 1974. Stochastic programs with fixed recourse the equivalent deterministic program. SAIM 16, 309–337.
Zhang, Xianjun, Sharma, R., He, Yanyi, 2012. Optimal energy management of a rural microgrid system using multi-objective optimization. In: Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES, Jan 2012, pp. 1–8.